The Limits of Direct Satellite Tracking With the Global Positioning System (GPS)

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Recent advances in high precision differential GPS-based satellite tracking can be applied to the more conventional direct tracking of low earth satellites. To properly evaluate the limiting accuracy of direct GPS-based tracking, it is necessary to account for the correlations between the a priori errors in GPS states, Y-bias, and solar pressure parameters. These can be obtained by careful analysis of the GPS orbit determination process. The analysis indicates that sub-meter accuracy can be readily achieved for a user above 1000 km altitude, even when the user solution is obtained with data taken 12 hours after the data used in the GPS orbit solutions.

I. Introduction

In recent years, a variety of differential GPS techniques have been described which promise to deliver decimeter accuracy in tracking low earth satellites [1] – [4]. Briefly, the high precision differential GPS techniques involve (1) simultaneous observation of the GPS satellites by a network of 6 to 10 ground receivers and by the user satellite; and (2) simultaneous estimation of GPS satellite states and clocks, the user state, and all receiver clocks.

Because of this need for a set of simultaneous GPS observations from both the user vehicle and a global ground network, the high precision differential techniques are somewhat cumbersome and are ill suited to near-real-time autonomous navigation. For this reason, we have investigated the limiting performance of direct (nondifferential) GPS-based tracking of low earth satellites, incorporating into this analysis a number of refinements that have emerged in recent years from differential GPS development work.

With direct GPS-based tracking (which in fact is the more conventional approach), the user position and the time offset from GPS time are obtained using measurements from only the user receiver. The collected GPS metric data, which include pseudorange and carrier phase data, are used together with information about the GPS orbits and clocks provided to the user separately, usually through the GPS broadcast data message. Highly accurate direct GPS-based tracking therefore requires highly accurate predetermination of GPS positions and time offsets. This predetermination of orbits and clocks is carried out using an extensive set of ground-based observations from a global tracking network.

In this respect, differential and direct tracking are rather similar: both require data from an on-board receiver and a global ground network. The key difference is that with direct tracking, the ground data are reduced independently, some hours in advance, to predetermine the GPS ephemeris and clock offsets. The central result of this article is that in making

this separation between ground data processing and on-board data processing, surprisingly little of the advantage of true differential tracking need be lost. To see this, we need to understand the correlations arising in the GPS orbit determination process and incorporate these into the analysis of the final user orbit error. Failure to do so can result in a substantial overestimation of the user error. This fact has generally been overlooked in previous analyses of direct GPS-based tracking.

Operationally, high precision direct GPS-based tracking might work as follows:

(1) A worldwide network of GPS ground receivers, such as the current Air Force Monitor Stations or the network being established for the differential GPS-based tracking demonstration of NASA's Ocean Topography Experiment (Topex) [1], is used to determine GPS orbits and clock parameters with sub-meter accuracy several hours in advance of user tracking.

(2) The GPS states and clock information are then propagated forward several hours, with accuracies that project to roughly 1 m and 10 ns at the time of user tracking, and are transmitted to the user satellite. Note that GPS clock accuracy degrades more than position accuracy in this process.

(3) The user satellite collects GPS measurements over a relatively short arc, typically 2 to 4 hours, to solve for its position and time offset from GPS time, as well as GPS clock offsets from one GPS reference clock. This requires a dynamic solution strategy which can be accurate to sub-meter levels only above 800 to 1000 km where gravity and drag model errors are sufficiently small.

II. Analysis

Using covariance analysis, we have evaluated the expected accuracy of the direct user orbit solution and the nature of the error sources. In order to explore the limiting accuracy of this technique we have assumed use of the high precision receivers, measurement calibration techniques, and geophysical models that have recently been developed for high precision GPS-based geodesy and differential satellite tracking. The dependence of user orbit accuracy on such factors as data arc length, the time interval between the end of the ground data arc and the beginning of the user data arc, and the data type used has also been studied. For comparison, results from a differential solution are also presented. The Topex satellite, which is scheduled to be launched in 1991 into a 1334-km circular orbit and which will carry a high performance GPS receiver, is assumed for the purposes of this analysis to be the test user

satellite. Note that real-time on-board precision orbit determination is beyond the scope of the Topex mission.

In addition to the assumed state-of-the-art precision of the tracking system, one feature distinguishes this analysis from previous studies of direct GPS-based tracking. Analyses of direct GPS-based tracking have generally treated the a priori errors in the components of GPS satellite states—errors which often dominate the final user error—as statistically independent. That is, the errors in the GPS state components have generally been represented by diagonal covariance matrices, although it is well known that important correlations between component errors exist. The usual reasons for this simplifying assumption are that the full GPS covariance matrices are unavailable or unknown, the analysis software can accept only a diagonal matrix, or both. In this study, by first analyzing the GPS satellite orbit determination process, we have been able to generate accurate full error covariance matrices for the GPS ephemerides supplied to the user and to employ these in the user orbit error analysis. The result is that the true user orbit error is seen to be consistently, and in most cases substantially, lower than the simpler analysis indicates.

All cases examined, both differential and direct, assume a worldwide network of six ground stations (in California, Spain, Brazil, Australia, Japan, and South Africa) and a full constellation of 18 GPS satellites.

For the differential solution, pseudorange measurements were assumed to be taken every five minutes from each ground station to all observable GPS satellites above a 10-degree elevation cutoff over a period of 24 hours starting at 1400 hours on March 21, 1986. Typically, 5 to 7 GPS satellites are visible at one time from each ground site. Topex was assumed to observe all GPS satellites above a 90-degree zenith angle for only the last two hours of this period. Measurement assumptions included 5 cm pseudorange every 5 minutes, a 10-degree elevation cutoff at the ground stations, and a zero-degree elevation cutoff for Topex. Note that pseudorange is assumed to be smoothed over the entire 5-minute measurement interval in order to achieve 5 cm precision. This level of performance is being routinely achieved by the "Rogue" geodetic receiver now undergoing field testing at the Jet Propulsion Laboratory (JPL) [5].

For the base case direct solution, only the two hours of measurements made by Topex were used. The GPS states and associated solar pressure parameters (including a Y-bias), obtained with a preliminary solution using a subset of the 24-hour ground track, were left unadjusted, and the effects of errors in those parameters were considered. In order to obtain a realistic covariance matrix for the GPS state and related force parameters used in the consider analysis, the first 20

hours of ground data from the 24-hour arc were used to solve independently for the GPS states, Y-bias, and solar pressure. The Y-bias parameter is associated with forces due to unmodeled thermal effects and solar panel misalignments [6].

The covariance matrix was assembled as follows: the 20-hour solution produced a computed covariance matrix for the GPS states, Y-bias, and solar pressure at epoch; this covariance matrix depends only upon the random data noise assumed in the solution. A consider analysis was then performed to evaluate the effects of troposphere and station location error upon the GPS states, Y-bias, and solar pressure, producing a second covariance matrix due to the consider variables. The two covariance matrices were then summed to form a single covariance matrix, containing computed error plus consider error, for use in the analysis of the Topex direct solution.

The covariance matrix for GPS states revealed 3-D position errors of about 1 meter when projected into the two-hour Topex data arc. This is compatible with the results of recent high precision GPS orbit determination demonstrations conducted under somewhat less favorable conditions [7], [8]. In fact, in this analysis the estimated error in Y-bias and solar pressure was somewhat higher than that obtained with multi-day data arcs in the same demonstration program.

The details of the measurement schedules for the three solutions—base case Topex direct, Topex differential, and GPS a priori state—are shown in Fig. 1. In both the GPS satellite solutions and the Topex differential solution, white noise clock models were used, producing independent clock solutions at each time point with one of the ground clocks used as reference. This is a general form of the double differencing technique that is widely used to eliminate receiver and satellite clocks as a source of error. In the Topex direct solution, where white noise modeling of all clocks is impossible, only the Topex clock was modeled as white noise, and the GPS clocks were modeled as quadratic functions with one GPS clock serving as reference. Extensive experience with real GPS data has shown that the highly stable GPS atomic clocks can be modeled as quadratic functions over many hours with sub-centimeter accuracy. (This situation will improve only when the more advanced Block II GPS satellites are deployed in the next few years.)

For the direct solution, the three quadratic coefficients (constant bias, rate, and rate–rate) were assumed to be known to 10 ns, 10 ns/(4 hours), and 10 ns/(4 hours)^2, respectively. These are in fact rather conservative values; solutions for actual GPS clocks with recent field data have been substantially better [9]. Ground receiver location errors were considered at 5 cm per component in the differential solution. The zenith troposphere error was considered at 1 cm, an accu-

racy that requires use of high performance water vapor radiometers at the ground sites. Finally, the earth gravity model was considered as an error source for Topex in both the differential and direct solutions. For this the covariance matrix associated with the 8 X 8 portion of the PGS3012 gravity model, produced recently by the Goddard Space Flight Center, was used. The higher order terms in the gravity covariance do not contribute significantly to the Topex position error for the short 2-hour data arc.

III. Results

A. Base Case

In the middle of the 2-hour Topex data arc, the estimated altitude errors are 17 cm for the base case direct solution and 7 cm for the differential solution. The RMS altitude errors over the arc are 21 cm and 11 cm. The errors as a function of time are shown in Fig. 2 for the direct solution and in Fig. 3 for the differential solution.

To test the importance of using the full a priori covariance matrix, a second consider analysis was conducted using only the diagonal elements of the covariance matrix for GPS states, Y-bias, and solar pressure. The resulting Topex position error grew to several meters. Clearly, the correlations between the consider parameters must be accounted for to accurately estimate final user errors.

B. Dependence on Topex Arc Length

In order to study the effect of data arc length on the Topex direct solution, a number of runs were made employing the same assumptions as the base case but differing in the length of the Topex data arc. Figure 4 shows a significant degradation in performance when the arc is reduced to 1 hour and 1/2 hour. Nevertheless, a 30-minute data arc gives an altitude accuracy better than 40 cm in the middle of the arc.

C. Dependence on Data Type

All cases examined so far have used precision pseudorange (5 cm data noise over 5 minutes). In this section we examine the direct solution method using carrier phase data alone (Fig. 5) and carrier phase together with pseudorange (Fig. 6). For these cases, carrier phase measurements with 0.5 cm data noise over 5-minute intervals were assumed. This noise level is consistent with the JPL "Rogue" receiver [5]. Other measurement assumptions were the same as for the base case. Figures 5 and 6 show very small computed errors and a slight improvement over the pseudorange base case in the middle of the data arc. The RMS errors for carrier and mixed data over the 2-hour data arc are 16 and 23 cm, respectively. The larger error in the mixed data type is anomalous and is due to nonoptimal

weighting of the data in the filter. With real data, the filter data weights could be adjusted by looking at the residuals to include the effects of mismodeling errors and to bring the total error with the mixed data type below that with either data type alone. Here the relative data weights are based only on the expected receiver data noise.

D. Dependence on the Time Offset From GPS Data

To study the effect of solving for GPS further in the past, the Topex data arc was progressively moved further away in 2-hour increments. In the base case the Topex data arc spanned March 22, 1400–1600. The next arc spanned 1600–1800, the next 1800–2000, and so on. The final arc in the series started 12 hours after the end of the arc used for the a priori GPS states. All other assumptions were the same as for the base case.

The errors due to gravity and the computed errors are similar for all of the five additional test arcs. Thus for comparison we show only the errors due to GPS state, Y-bias, and solar pressure. As expected, the solution degrades as we move further away from the solution for GPS. The degradation is not monotonic, however; for the arcs that are 4 and 6 hours away, performance is actually better than the base case that is only 2 hours away from the solution for GPS states. This is consistent with the periodic signature in GPS orbit errors in addition to the general trend of increasing error further from the data arc. Figure 7 shows the RMS errors as a function of the time interval between the end of the GPS data arc and the beginning of the user data arc. Figure 8 shows the errors over the

2-hour Topex data arc at 2, 6, and 12 hours from the GPS data arc. Even after an interval of 12 hours, the RMS error due to GPS state, Y-bias, and solar pressure over the arc is only about 30 cm.

IV. Conclusions

Direct GPS-based tracking of low earth orbiters can deliver sub-meter accuracy at altitudes above roughly 1000 km, with the user tracking up to 12 hours after the GPS orbit solutions. The major error sources are the predetermined GPS states, Y-bias, solar pressure, and the model for the earth's gravity field. In order to obtain a realistic estimate of the accuracy that can be achieved, a full covariance matrix must be used to represent the a priori error in the pre-adjusted parameters.

At altitudes lower than 1000 km, significant errors in the force models, principally the models for the gravity field and atmospheric drag, will begin to corrupt the direct dynamic solution. Therefore, to maintain high accuracy at lower altitudes, nondynamic (geometric or kinematic) or reduced dynamic strategies must be adopted. Such techniques have been evaluated extensively in connection with differential satellite tracking and promise to deliver sub-decimeter accuracy down to the lowest possible altitudes [10], [11]. Differential tracking, however, requires simultaneous GPS observations by the user and a global network of ground receivers. It remains to investigate the potential performance of the nondynamic and reduced dynamic techniques in direct GPS-based satellite tracking.

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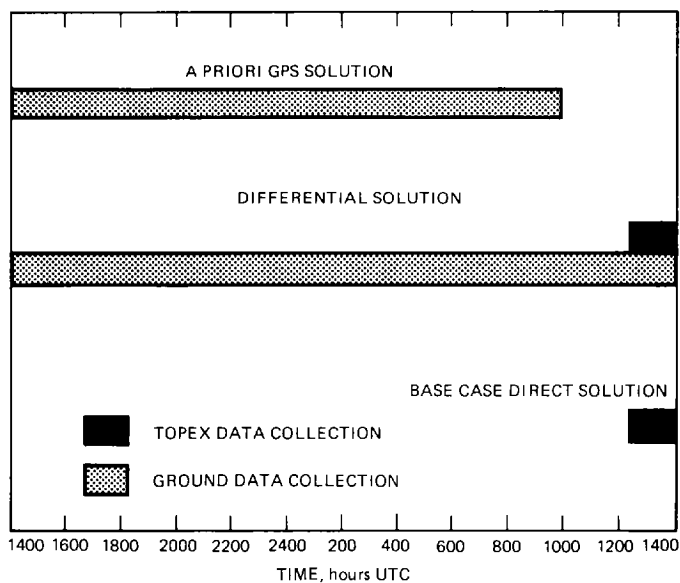
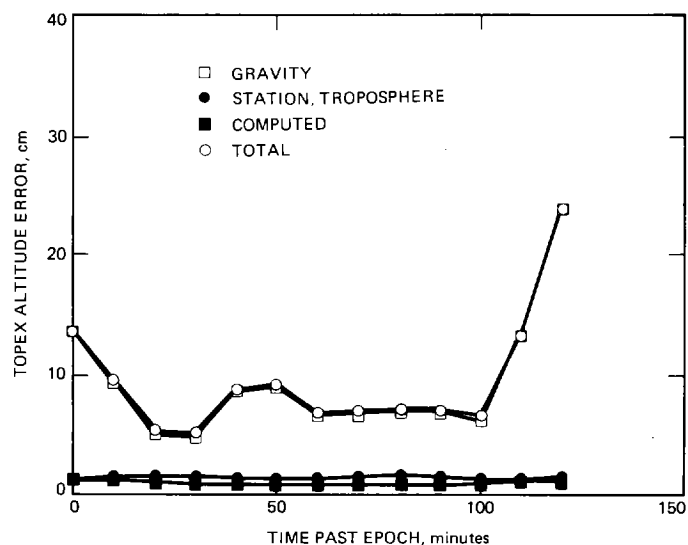**Fig. 1. Basic measurement scenario, March 21–22, 1986**

Fig. 3. Differential solution, 2-hour Topex arc, 24-hour GPS arc, pseudorange
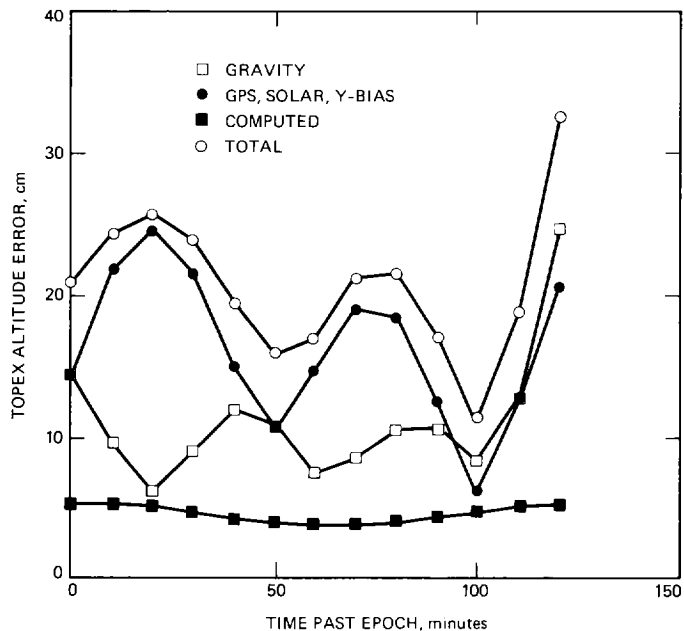

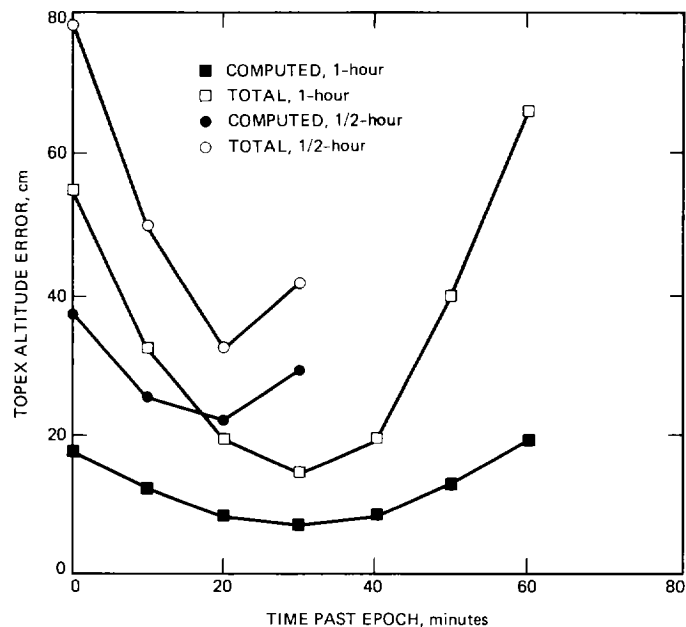**Fig. 2. Base case direct solution, 2-hour arc, pseudorange**

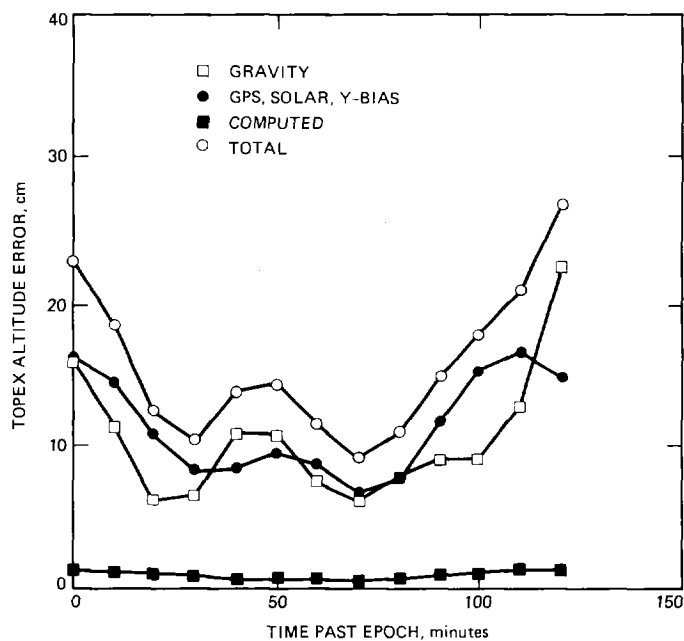**Fig. 4. Accuracy dependence on Topex arc length**
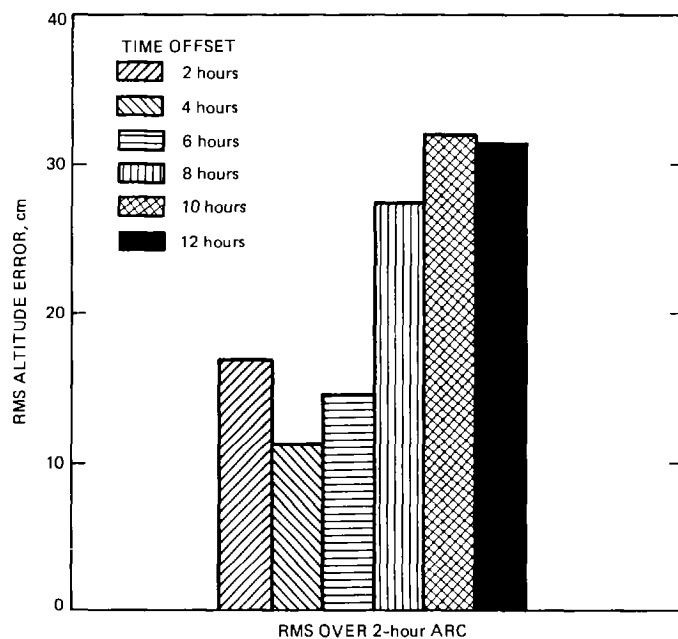
Fig. 5. Direct solution with carrier phase data

Fig. 7. Variation of RMS Topex altitude error with the time offset from GPS solution arc
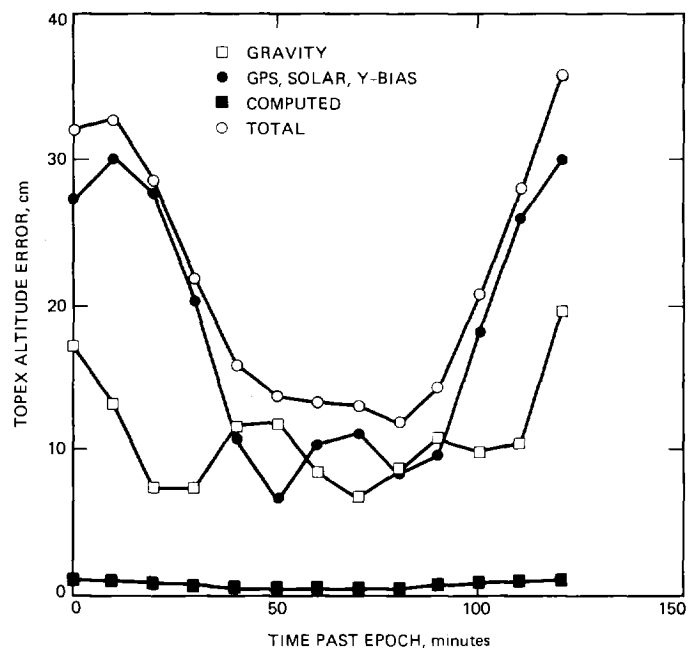

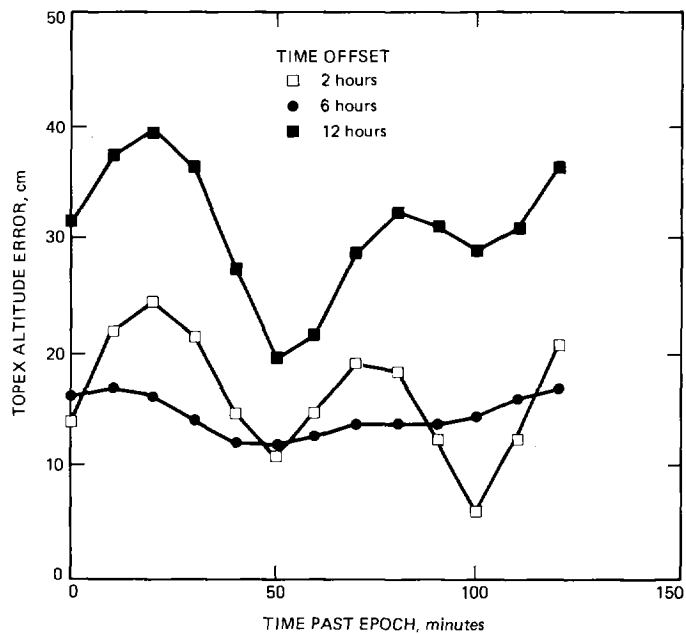**Fig. 6. Direct solution with mixed data types**

Fig. 8. Errors over a 2-hour period for different time offsets from GPS solution